Axioms of Probability Lecture 5

A short Probability Quiz

1) A closet contains 5 pairs of shoes. If 3 shoes are randomly selected, what is the probability of

(a) no complete pair

(b) Exactly 1 complete pair

Solution: Assign each shoe a population ID 1-10.

The sample space can then be the set of all possible subsets of $\{1, \ldots, 10\}$ of size 3.

(a) Assign pair numbers 1-5. Within each pair, designate by R and L the left and right shoe respectively.

The Kafka protocol for E = no complete pair could be

$\boxed{1,\ 3,\ 5}$	$\boxed{L \mid L \mid R}$
pair numbers	shoe type
$\uparrow$	$\uparrow$
$\binom{5}{3}$	2^3

Hence $P(E) = \dfrac{\binom{5}{3} 2^3}{\binom{10}{3}} = \dfrac{5 \cdot 4 \cdot 3 \cdot 2^3}{10 \cdot 9 \cdot 8}$

$$= \frac{2}{3}$$

(b) A = Exactly one complete pair

clearly $p(S) = p(A \cup E) = p(A) + p(E)$. Hence

$p(A) = 1 - \frac{2}{3} = \frac{1}{3}$.

Alternatively, the Kafka protocol for A could be

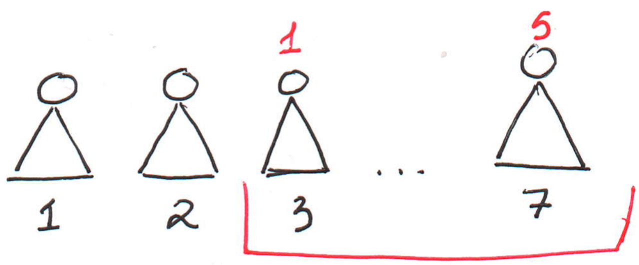
$$5 \longrightarrow \boxed{} \quad \boxed{} \longleftarrow 8$$

pair num. rem. shoe

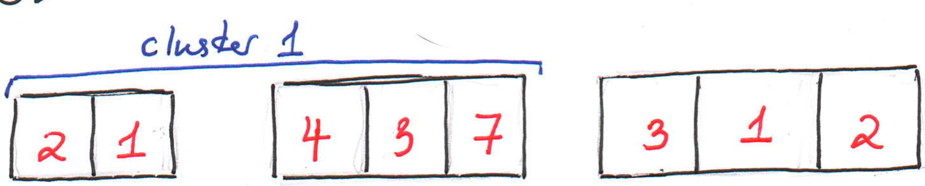
Thus $P(A) = \dfrac{5 \cdot 8}{\binom{10}{3}} = \dfrac{5 \cdot 8 \cdot 3!}{10 \cdot 9 \cdot 8} = \dfrac{1}{3}$

2) 7 students $\{1-7\}$ are seated in a row at random. What is the probability that exactly 3 students are seated between student #1 and student #2?

<u>Solution:</u>



Let E = event 3 students between #1 and #2.



1 − 2 or 2 − 1 ↑ 2

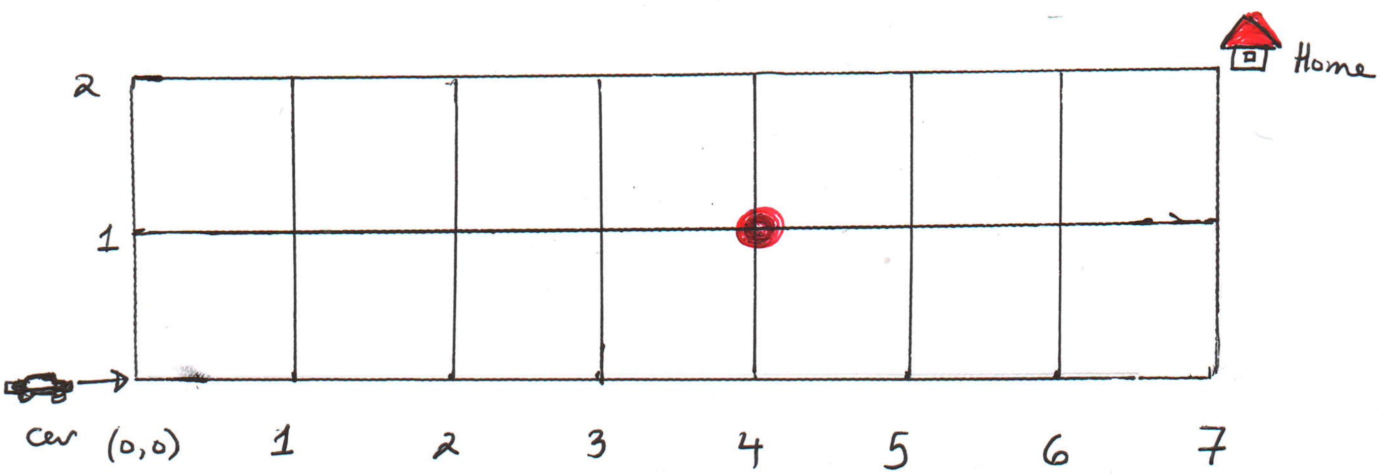
wedged st. ↑ $3!\binom{5}{3}$

place of cluster 1 & single sts. ↑ $3!$

Thus $P(E) = \dfrac{2 \cdot 3! \binom{5}{3} \cdot 3!}{7!}$

$$= \frac{2 \cdot 3! \cdot \cancel{5} \cdot \cancel{4} \cdot 3}{7 \cdot 6 \cdot \cancel{5!}} = \frac{1}{7}$$

3) A New York driver is racing through a school zone at the neck-breaking speed of 26 mi/hr



If his house is located at (7,2) and the trap camera is in position (4,1), what is his probability of avoiding a fine? Assume that at each intersection, the driver randomly chooses to whether to move up or stay along a horizontal path.

<u>Solution:</u> Let F be the event that he gets a fine. To get home, driver must execute $7+2$ motions of which 2 must be vertical. Thus our sample space S has $\binom{7+2}{2} = \binom{9}{2}$ outcomes.

To get a fine, driver must pass the point $(4,1)$. To do that, driver must make one up motion in the first $4+1$ blocks followed by 1 up motion in the next $3+1$ blocks after passing $(4,1)$.

$$\text{Thus } P(F) = \frac{5 \cdot 4}{\binom{9}{2}} = \frac{2 \cdot 5 \cdot 4}{9 \cdot 8} = \frac{5}{9}$$

The probability of not getting a ticket is therefore $1 - P(F) = \frac{4}{9} = 0.\overline{4} \approx 44\%$